

Review Notes - Solving Quadratic Equations

$$ax^2 + bx + c = 0$$

What does solve mean?

★ FIND ALL VALUES THAT MAKE THE SENTENCE TRUE!

★ How many solutions do we expect?

Degree = 2 \rightarrow Up to 2 solutions

Methods for Solving Quadratic Equations:

Solving by using Square Roots

Solving by Factoring using the Zero Product Property

Solving by Quadratic Formula

Solving Quadratic Equations Using Square Roots - Part 1

Definition: If b is a square root of a , then $b^2 = a$.

Example: 4 is a square root of 16 since $4^2 = 16$.

What are the square roots of 16?

Another way to ask the same question is

"What number squared is 16?"

$$(4)^2 = 16 \quad \text{and} \quad (-4)^2 = 16$$

There are two answers to this question!!!

16 has two square roots, 4 and -4. 4 is called the positive square root and -4 is the negative square root.

The Radical Symbol: $\sqrt{\quad}$ $\sqrt[2]{\quad}$ $\sqrt[3]{\quad}$

[The Radical symbol is a grouping symbol.]

$\sqrt{9}$ "The positive square root of 9."

$-\sqrt{9}$ "The negative square root of 9."

$\pm\sqrt{9}$ "The positive and negative square roots of 9."

Every positive number has two(2) square roots.

Every negative number has zero(0) square roots.

Zero has one(1) square root.

Simplify each expression.

Ex 1: $\sqrt{25}$

5

Ex 2: $-\sqrt{64}$

-8

Ex 3: $3\sqrt{49}$

3(7)

21

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Ex 4: $-\sqrt{100} + \sqrt{16}$

$-10 + 4$

-6

Ex 5: $6 \pm \sqrt{121}$

6 ± 11

$6-11$ or $6+11$

$-5, 17$

Ex 6: $-3 \pm 2\sqrt{9}$

$-3 \pm 2(3)$

-3 ± 6

$-3-6$ or $-3+6$

$-9, 3$

Ex 7: $\frac{-1 \pm \sqrt{25}}{2}$

2

$\frac{-1 \pm 5}{2}$

$\frac{-1-5}{2}$ or $\frac{-1+5}{2}$

$-3, 2$

Ex 8: $\frac{-3 \pm \sqrt{3^2 - 4(1)(2)}}{2(1)}$

$2(1)$

$\frac{-3 \pm \sqrt{9-8}}{2}$

2

$\frac{-3 \pm \sqrt{1}}{2}$

2

$\frac{-3 \pm 1}{2}$

2

$-2, -1$

Ex 9: $\frac{7 \pm \sqrt{(-7)^2 - 4(6)(-5)}}{2(6)}$

$2(6)$

$\frac{7 \pm \sqrt{49+120}}{12}$

12

$\frac{7 \pm \sqrt{169}}{12}$

12

$\frac{7 \pm 13}{12}$

12

$\frac{-6}{12}$ or $\frac{20}{12}$

$-\frac{1}{2}, \frac{5}{3}$

Ex 10:
$$\frac{5 \pm \sqrt{(-5)^2 - 4(2)(-3)}}{2(2)}$$

$$\frac{5 \pm \sqrt{25 + 24}}{4}$$

$$\frac{5 \pm \sqrt{49}}{4}$$

$$\frac{5 \pm 7}{4}$$

$$-\frac{2}{4} \text{ or } \frac{12}{4}$$

$$\boxed{-\frac{1}{2}, 3}$$

Solving Quadratic Equations Using Square Roots - Part 2

Simplest Radical Form - SRF

An expression is in Simplest Radical Form if it meets the following conditions.

1. No radicands have perfect square factors other than one. [inside the radical symbol]
2. No radicands contain fractions.
3. No radicals in the denominator.

$$\sqrt{23} \checkmark$$

$$\sqrt{700}$$

$$\frac{4}{\sqrt{10}}$$

$$\sqrt{\frac{2}{5}}$$

We use the following properties to simplify radicals.

1. **Product Property**

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$$

2. **Quotient Property**

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

3. **Multiplicative Identity**

$$\frac{a}{\sqrt{b}} \cdot \frac{\sqrt{b}}{\sqrt{b}} = \frac{a\sqrt{b}}{\sqrt{b^2}} = \frac{a\sqrt{b}}{b}$$

Write each expression in SRF **and** approximate to the nearest hundredth.

Ex 1: $\sqrt{20}$

$$\sqrt{4} \sqrt{5}$$

$$\boxed{2\sqrt{5}}$$

Ex 2: $\sqrt{72}$

$$\sqrt{36} \sqrt{2}$$

$$\boxed{6\sqrt{2}}$$

Ex 3: $-\sqrt{300}$

$$-\sqrt{100} \sqrt{3}$$

$$\boxed{-10\sqrt{3}}$$

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Ex 4: $\sqrt{\frac{12}{25}}$

$$\frac{\sqrt{12}}{\sqrt{25}}$$

$$\frac{\sqrt{4}\sqrt{3}}{5}$$

$$\boxed{\frac{2\sqrt{3}}{5}}$$

Ex 5: $\sqrt{\frac{60}{49}}$

$$\frac{\sqrt{4}\sqrt{15}}{\sqrt{49}}$$

$$\boxed{\frac{2\sqrt{15}}{7}}$$

Ex 6: $\sqrt{\frac{4}{3}}$

$$\frac{\sqrt{4}}{\sqrt{3}}$$

$$\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\frac{2\sqrt{3}}{\sqrt{9}}$$

$$\boxed{\frac{2\sqrt{3}}{3}}$$

Ex 7: $\sqrt{\frac{27}{8}}$

$$\frac{\sqrt{9}\sqrt{3}}{\sqrt{4}\sqrt{2}}$$

$$\frac{3\sqrt{3}}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$\frac{3\sqrt{6}}{2\sqrt{4}}$$

$$\boxed{\frac{3\sqrt{6}}{4}}$$

Ex 8: $\sqrt{12} \cdot \sqrt{3}$

$$\sqrt{36}$$

$$\textcircled{6}$$

Ex 9: $\sqrt{40} \cdot \sqrt{15}$

$$\sqrt{600}$$

$$\sqrt{100}\sqrt{6}$$

$$\boxed{10\sqrt{6}}$$

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<p>Ex 10: $(3\sqrt{11})^2$ $(3\sqrt{11})(3\sqrt{11})$ $9\sqrt{121}$ $9(11)$ (99)</p>	<p>Ex 11: $\frac{4 \pm \sqrt{40}}{2}$ $\frac{4 \pm \sqrt{4}\sqrt{10}}{2}$ $\frac{4 \pm 2\sqrt{10}}{2}$ $2 \pm \sqrt{10}$</p>	<p>Ex 12: $\frac{-6 \pm \sqrt{27}}{3}$ $\frac{-6 \pm \sqrt{9}\sqrt{3}}{3}$ $\frac{-6 \pm 3\sqrt{3}}{3}$ $-2 \pm \sqrt{3}$</p>
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Ex 13: $\frac{-8 \pm \sqrt{(8)^2 - 4(1)(-2)}}{2(-2)}$

$\frac{-8 \pm \sqrt{64 + 8}}{-4}$

$\frac{-8 \pm \sqrt{72}}{-4}$

$\frac{-8 \pm \sqrt{36}\sqrt{2}}{-4}$

$\frac{-8 \pm 6\sqrt{2}}{-4}$

$[GCF = -2]$

$\frac{4 \pm 3\sqrt{2}}{2}$

Solving Quadratic Equations Using Square Roots - Part 3

$0 = ax^2 + bx + c$ can be solved using square roots if $b = 0$.

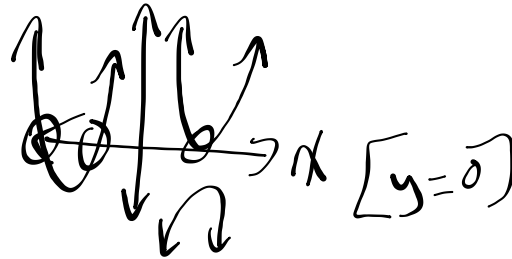
This means. $0 = ax^2 + c$ ←

What does solve mean?

★ FIND ALL VALUES THAT MAKE THE SENTENCE TRUE!

★ How many solutions do we expect?

Degree is 2 → 2 solutions



Solving Quadratic Equations Using Square Roots

$0 = ax^2 + bx + c$ can be solved using square roots if $b = 0$.

This means $0 = ax^2 + c$.

Write solutions in SRF. [Isolate x^2 .]

Ex 1: $x^2 - 4 = 0$

$$x^2 = 4$$

$$x = \pm\sqrt{4}$$

$$x = \pm 2$$

Ex 2: $x^2 - 64 = 0$

$$x^2 = 64$$

$$x = \pm\sqrt{64}$$

$$x = \pm 8$$

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Ex 3: $x^2 - 16 = 20$

$$x^2 = 36$$

$$x = \pm \sqrt{36}$$

$$x = \pm 6$$

Ex 4: $x^2 + 9 = 59$

$$x^2 = 50$$

$$x = \pm \sqrt{50}$$

$$x = \pm \sqrt{25} \sqrt{2}$$

$$x = \pm 5\sqrt{2}$$

Ex 5: $x^2 + 19 = 10$

$$x^2 = -9$$

$$x = \pm \sqrt{-9}$$

No Real Solution

Ex 6: $3x^2 - 8 = 13$

$$3x^2 = 21$$

$$x^2 = 7$$

$$x = \pm \sqrt{7}$$

Ex 7: $12 - 5x^2 = -28$

$$-5x^2 = -40$$

$$x^2 = 8$$

$$x = \pm \sqrt{8}$$

$$x = \pm \sqrt{4} \sqrt{2}$$

$$x = \pm 2\sqrt{2}$$

Ex 8: $-15 + 3x^2 = 5$

$$3x^2 = 20$$

$$x^2 = \frac{20}{3}$$

$$x = \pm \sqrt{\frac{20}{3}}$$

$$x = \pm \frac{\sqrt{4} \sqrt{5}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$x = \pm \frac{2\sqrt{15}}{3}$$

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Simplify each radical expression. Circle your final answer.

1. $\sqrt{80}$
 $\sqrt{16} \sqrt{5}$
 $4\sqrt{5}$

2. $-\sqrt{125}$
 $-\sqrt{25} \sqrt{5}$
 $-5\sqrt{5}$

3. $\pm\sqrt{120}$
 $\pm\sqrt{4} \sqrt{30}$
 $\pm 2\sqrt{30}$

Simplify each radical expression.
 Circle your final answer.

4. $\sqrt{\frac{5}{4}}$
 $\frac{\sqrt{5}}{\sqrt{4}}$
 $\frac{\sqrt{5}}{2}$

5. $\sqrt{\frac{2}{27}}$
 $\frac{\sqrt{2}}{\sqrt{9} \sqrt{3}}$
 $\frac{\sqrt{2}}{3\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$
 $\frac{\sqrt{6}}{3(3)}$
 $\frac{\sqrt{6}}{9}$

6. Solve and write your solution in SRF.

$$4x^2 - 15 = 60$$

$$4x^2 = 75$$

$$x^2 = \frac{75}{4}$$

$$x = \pm \sqrt{\frac{75}{4}}$$

$$x = \pm \frac{\sqrt{25} \sqrt{3}}{\sqrt{4}}$$

$$x = \pm \frac{5\sqrt{3}}{2}$$